Multi-Mbar Phase Transitions in Minerals

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INTRODUCTION

MgSiO₃ perovskite is the most abundant mineral in the Earth’s lower mantle. A structural phase transition in this phase to a CaIrO₃-type polymorph was discovered in 2004 (Murakami et al. 2004; Oganov and Ono 2004; Tsuchiya et al. 2004). This new polymorph, the so-called post-perovskite (PPV) phase, was produced at pressures and temperatures close to those expected at the core-mantle boundary, 125 GPa and 2,500 K (Murakami et al. 2004). In the Earth, the PPV phase is the final form of MgSiO₃. This surprising discovery invited a new question: what is the next polymorph of MgSiO₃? MgSiO₃ PPV consists of SiO₃ layers intercalated by magnesium (Fig. 1). Therefore, it is natural to expect still other pressure induced transitions to more isotropic close-packed looking structures. This question has acquired further importance since the discovery of terrestrial-type exoplanets: the Super-Earth planet with ~7 Earth masses, GJ876d (Rivera et al. 2005), and the Saturn-like planet with a massive dense core with ~67 Earth masses, D149026b (a dense-Saturn) (Sato et al. 2005). Many others have been found since then. Pressures and temperatures in the mantle of these planets are much higher than in the Earth. There is also a pressing need to understand and model matter in the core of the giants, particularly the solar ones, Jupiter, Saturn, Uranus, and Neptune. In GJ876d, pressure and temperature at its core-mantle boundary was roughly estimated to be ~1 TPa (10 Mbar) and ~4,000 K (Valencia et al. 2006). The gas giants, Jupiter and Saturn, and the icy giants, Uranus and Neptune, have small dense cores surrounded by hydrogen/helium and ice, respectively. Pressures and temperatures at the core-envelope boundaries of these planets have been estimated to be 40 Mbar and 15,000~20,000 K in Jupiter, 10 Mbar and 8,500~10,000 K in Saturn, 8 Mbar and ~8,000 K in Uranus and Neptune (Guillot 2004). To improve modeling of their interiors, there should be a better understanding of possible...
phases (MgSiO$_3$ PPV, etc.) and their equations of state under these extreme conditions. Experimentally, it is very challenging to achieve such extremely high pressures and temperatures. The National Ignition Facility (NIF), expected to enter in operation in 2010, offers hope for some data on materials at these conditions. However, first-principles computational methods are very powerful and promising to investigate materials properties at these extreme conditions. They are forging ahead and making predictions to be tested at NIF in the 10$^5$ Mbar and 10$^5$ K regime (Umemoto et al. 2006a; Wu et al. 2008; Sun et al. 2008; Umemoto et al. 2008).

In this paper, we discuss first-principles investigations of post-PPV transitions in planet-forming minerals, MgSiO$_3$ and Al$_2$O$_3$, at multi-Mbar pressures. We will also discuss low-pressure analogs of these minerals. They can provide experimentally convenient alternatives for exploration of post-PPV transitions. We will see a close relationship between the multi-Mbar chemistry of planet-forming minerals and those of rare-earth sesquisulfides and transition-metal sesquioxides.

**COMPUTATIONAL BACKGROUND**

This section briefly describes these first principles density functional theory (DFT) (Hohenberg and Kohn 1964; Kohn and Sham 1965) calculations. DFT makes use of approximations for the exchange-correlation (XC) energy. Several XC energy functionals exist, but the most widely used are based on the local-density approximation (LDA) (Ceperley and Alder 1980; Perdew and Zunger 1981) and on the generalized density approximation (GGA), in particular the PBE functional (Perdew et al. 1996). They have been successfully used for many minerals (Wentzcovitch et al. 2010, in this volume). PBE-GGA is also relatively successful for calculations in H$_2$O-ice. Wave functions are expanded in plane waves in combination with pseudopotentials (Vanderbilt 1990). At multi-Mbar pressures, care must be exercised when generating pseudopotentials. Inter-atomic distances become very small at these extreme pressures, and pseudopotential cut-off radii must be sufficiently small to avoid core overlap. It is often necessary to promote semi-core state to valence. In Umemo et al. (2006a), 2$s$ and 2$p$ states of silicon, usually treated as core states, were promoted to valence states. Using the plane wave pseudopotential method, total energy, forces, and stress are efficiently evaluated. Structural search and optimizations are performed at arbitrary pressures using a damped form of variable-cell-shape molecular dynamics (Wentzcovitch 1991; Wentzcovitch et al. 1993). The dynamical stability of optimized structures is assessed by calculation of phonon frequencies. These are calculated by diagonalizing the dynamical matrices obtained using density-functional perturbation theory (Giannozzi et al. 1991; Baroni et al. 2001). The system is dynamically stable if all phonon frequencies are real throughout the Brillouin zone. Phonon frequencies are also used to calculate the Helmholtz free energy within the quasi-harmonic approximation (QHA) (Wallace 1972). All thermodynamic properties, including Gibbs free energies at arbitrary pressures and temperatures, can then be obtained. Knowledge of the Gibbs free energies of two or more phases, phase transformations can be investigated.

**DISSOCIATION OF MgSiO$_3$ PPV**

The first prediction of a post-PPV transition was the dissociation of MgSiO$_3$ (Umemoto et al. 2006a) into CsCl-type MgO and cotunnite-type SiO$_2$ at 1.12 TPa (11.2 Mbar) in static enthalpy calculations (Fig. 2). CsCl-type MgO and cotunnite-type SiO$_2$ have not been synthesized experimentally yet, but they are the most probable candidates of high-pressure forms of MgO and SiO$_2$ in the dissociation pressure range. The calculated phase boundary shown in Figure 3 indicates that MgSiO$_3$ PPV should not exist any longer in the cores of the gas giants (Jupiter and Saturn) but could survive in cores of the icy giants (Uranus and Neptune). This transition may occur in the mantles of Super-Earths-type exoplanets (Rivera et
depending on their masses and temperatures, and in dense larger exoplanets (Sato et al. 2005). Along the dissociation phase boundary the mixture of products was found to be denser than the PPV phase by 1–3%, depending on temperature. The coordination number of silicon increases from 6 to 9 and the averaged bond lengths increase as well. This lowers the vibrational entropy of the dissociation products. Consequently, the Clapeyron slope for this transition is negative. In GJ876d, the dissociation is likely to occur near its CMB (Valencia et al., 2006). The eventual occurrence of this endothermic transition with a large negative Clapeyron slope would be equivalent to the occurrence of the endothermic post-spinel transition near the core of Mars. Geodynamical modeling suggests that this might be the cause of a proposed large martian superplume (Weinstein et al., 1995). Convection in D149026b (Sato et al. 2005), where internal pressures and temperatures should be much higher than in Saturn,
could be dramatically affected. A transformation with such large negative Clapeyron slope in the middle of its silicate core-mantle is likely to inhibit convection (Tackley 1995), promote layering, and produce a differentiated core-mantle, with the bottom layer consisting primarily of oxides. Thermal excitation of carriers at the high temperatures relevant for the solar giants and exoplanets affects noticeably important mineral properties. Finite electronic temperature (Mermin 1965; Wentzcovitch et al. 1992) calculations indicated that although the dissociation products are intrinsic semiconductors with electronic band gaps, the carrier concentrations in cotunnite-type SiO2 become typical of semimetals or heavily-doped semiconductors at 1 TPa and 10,000–20,000 K (Umemoto et al. 2006a). Hence these minerals can be seen essentially as metals with rather high electric and thermal conductivities. This is important information for improving models of the solar giants’ interiors and of terrestrial exoplanets.

**LOW-PRESSURE ANALOG OF MgSiO3**

Although the dissociation of MgSiO3 should be important for the solar giants and terrestrial exoplanets, the predicted dissociation pressure is still too high to be achieved routinely by experiments (~1 TPa). Therefore, low-pressure analogs of MgSiO3 are highly desirable for experimental investigations of properties of the CaIrO3-type structure, including its dissociation. In general, analog compounds with the same structure and larger ions tend to have lower transition pressures. Atomic/ionic sizes increase downwards and towards the left in the periodic table. There are several candidates for low-pressure analogs of MgSiO3, e.g., CaIrO3 (Hirose and Fujita 2005; Tsuchiya and Tsuchiya 2007), MgGeO3 (Ross and Navrotsky 1988; Hirose et al. 2005; Kubo et al. 2006, 2008), Mn2O3 (Santillán et al. 2006), and NaMgF3 (Liu et al. 2005; Hustoft et al. 2008). Among them, NaMgF3 (neighborite) is one of the best candidates. It is a stable Pbnm perovskite phase at ambient conditions and undergoes a pressure-induced phase transition to the CaIrO3-type phase (Liu et al. 2005; Hustoft et al. 2008). Umemoto et al. (2006b) predicted by first principles that NaMgF3 has qualitatively the same phase diagram as MgSiO3: the PPV transition has a positive Clapeyron slope and the PPV dissociation into CsCl-type NaF and cotunnite-type MgF2 has a negative Clapeyron slope (Umemoto et al. 2006b). NaF and MgF2 are also low-pressure analogs of MgO and SiO2 (Yagi et al. 1983; Haines et al. 2001). The dissociation pressure in NaMgF3 occurs at ~40 GPa (Fig. 4), which is much lower than that of MgSiO3 and can be easily achieved by static compression.

![Figure 4. Calculated phase boundaries of NaMgF3 (Umemoto and Wentzcovitch 2006). Two dashed lines denote the metastable phase boundaries between CaIrO3-type and U2S3-type, and between U2S3-type and P63/mmc phases, respectively. The solid line in the upper left corner denotes the experimental phase boundary between orthorhombic perovskite and cubic phases (Zhao et al. 1994).](image-url)
experiments. Therefore, NaMgF$_3$ should be a good low-pressure analog of MgSiO$_3$ to test the possibility of a dissociation transition. The PPV transition and subsequent dissociation have also been predicted in CaSnO$_3$ (Tsuchiya and Tsuchiya 2006). Despite the latter being an oxide compound, its dissociation pressure was found to be ~70 GPa, which is also quite low compared to that in MgSiO$_3$.

**PREDICTION OF POST-PPV CRYSTALLINE PHASES**

Although the dissociations of MgSiO$_3$, NaMgF$_3$, and CaSnO$_3$ PPV were predicted, it is not guaranteed that all ABX$_3$ PPVs should dissociate. Some of them might prefer to undergo a post-PPV transition to another ABX$_3$ polymorph before dissociating. What could happen if the dissociation is inhibited? Umemoto and Wentzcovitch (2006) investigated the metastable compression of CaIrO$_3$-type NaMgF$_3$ beyond the predicted dissociation pressure (~40 GPa). Up to ~80 GPa, all lattice constants of the CaIrO$_3$-type NaMgF$_3$ decreased as usual. However, there were two anomalies in the behavior of the lattice constants under pressure: increases in lattice constants $a$ and $c$ at 80 GPa and an abrupt jump of all lattice constants at 150 GPa (Fig. 5).

The first anomaly is related to the softening of an acoustic mode at the zone-edge $Y$ point (Fig. 6). After superposing the atomic displacements of this particular soft mode to the structure followed by structural re-optimization, new bonds appeared between magnesium and fluorine in adjacent layers. This resulted in a phase transformation from the CaIrO$_3$-type phase to a phase with $Pmcn$ symmetry (the $Pnma$ symmetry in the standard setting), a sub-group of $Cmcm$. This phase is isostructural with U$_2$S$_3$ (Fig. 7b). The magnesium coordination number is 7, larger than that of the CaIrO$_3$-type phase, 6. A possible crystallographic relationship between the CaIrO$_3$-type and the U$_2$S$_3$-type structures was discussed by Hyde et al. (1979). It is interesting to note that UFeS$_3$ has the CaIrO$_3$-type structure. Replacement of iron by uranium leads to the U$_2$S$_3$-type structure. The second anomaly corresponds to an increase in symmetry of the CaIrO$_3$-type phase and generates a new phase with $P6_3/mmc$ symmetry, a super-group of $Cmcm$ (Fig. 7c). The magnesium coordination number in this phase is 8, higher than 6 in the CaIrO$_3$-type and 7 in the U$_2$S$_3$-type phases. The structural unit is no longer the MgF$_6$ octahedron but an MgF$_8$ parallelepiped. Parallelepipeds share edges to form layers in the $ab$ plane. Each layer contacts the adjacent ones at the parallelepipeds’ apices along the $[0001]$ direction. Sodium is located in interstitial sites between the MgF$_8$ parallelepipeds. The sodium coordination number is 11. Sodium and magnesium stack in an ABAC sequence (A: magnesium and B, C: sodium), i.e., the sub-lattice formed by sodium and magnesium has the NiAs structure. The sodium and fluorine sub-lattices have the IrAl$_3$-type structure (Hyde and Andersson 1989). As far as we know, the $P6_3/mmc$ phase has not been identified in any material experimentally so far. Both U$_2$S$_3$-type and $P6_3/mmc$ phases are dynamically stable phases of NaMgF$_3$. Therefore, these structures are two potential candidates.
for post-PPV transitions in NaMgF$_3$ instead of the dissociation (Umemoto and Wentzcovitch 2006). Although they are metastable with respect to the dissociation products, the U$_2$S$_3$-type phase could actually be observed experimentally by compressing CaIrO$_3$-type NaMgF$_3$ at sufficiently low temperatures. The energy barrier for the dissociation is expected to be higher than that for the U$_2$S$_3$-type transition which is related to a soft mode. In addition, at low temperature, the phase boundaries for the dissociation and for the U$_2$S$_3$-type transition are close to each other. Recently Martin et al. (2006) reported diamond anvil cell experiments in Cmcm NaMgF$_3$ showing new X-ray diffraction peaks with increasing pressure. However, these peaks could not be
attributed only to a mixture of \textit{Cmcm} and its dissociation products, NaF and MgF$_2$. The superposition of peaks seems to be considerable and the analysis of the published pattern is challenging and inconclusive; but, the new X-ray diffraction pattern may contain peaks related to the U$_2$S$_3$-type phase. In MgSiO$_3$, the static transition pressure to the U$_2$S$_3$-type phase is calculated to be beyond 1.6 TPa (Umemoto and Wentzcovitch 2006), which is considerably higher than the dissociation pressure of ~1 TPa. In nature these pressures are realized in the interior of the giant planets and exoplanets where temperatures are also expected to be very high, $10^{3.5}$-$10^4$ K. Since diffusion barriers are expected to be overcome in planetary time scales, dissociation into oxides is still the most likely pressure-induced transition in CaIrO$_3$-type MgSiO$_3$ in planetary interiors.

**POST-POST-PEROVSKITE TRANSITION IN Al$_2$O$_3$**

Two metastable candidate post-PPV structures—U$_2$S$_3$-type and \textit{P6$_3$/mmc} structures—were proposed for NaMgF$_3$ under pressure, but the most probable transition should be the dissociation into CsCl-type NaF and cotunnite-type MgF$_2$. However, sometimes dissociation is “forbidden.” Al$_2$O$_3$ (alumina) is expected to undergo a non-dissociative post-PPV transition. Its highest-pressure phase identified experimentally so far, the CaIrO$_3$-type, has unlikely dissociation products, AlO and AlO$_2$. For Al$_2$O$_3$, two phase transitions have been established so far: corundum—Rh$_2$O$_3$(II)-type (Cynn et al. 1990; Marton and Cohen 1994; Thomson et al. 1996; Funamori and Jeanloz 1997; Mashimo et al. 2000; Lin et al. 2004) and Rh$_2$O$_3$(II)-type—CaIrO$_3$-type (PPV) (Caracas and Cohen 2005; Oganov and Ono 2005; Tsuchiya et al. 2005; Ono et al. 2006). Al$_2$O$_3$ is an important compound in high-pressure technology and geophysics. It is used as window material in shock-wave experiments (McQueen and Isaak 1990). The pressure dependence of the fluorescence line of ruby, Al$_2$O$_3$ doped with chromium, serves as a pressure-marker in diamond-anvil-cell experiments (Mao and Bell 1976; Chen and Silvera 1996). In Earth and planetary sciences, Al$_2$O$_3$ is a major chemical component in solid solution with MgSiO$_3$ garnet, PV, and PPV. The formation of these solid solutions with Al$_2$O$_3$ changes the properties, phase boundaries, electrical conductivity, oxidation state, and spin states of iron in MgSiO$_3$, (e.g., Wood and Rubie 1998; Xu et al. 1998; Zhang and Weidner 1999; Frost et al. 2004; Li et al. 2004; Taneno et al. 2005; Nishio-Hamane et al. 2007).

Static enthalpy calculations by Umemoto and Wentzcovitch (2008) clearly showed a post-PPV transition in Al$_2$O$_3$ to a U$_2$S$_3$-type phase (Fig. 8). The static transition pressure was predicted to be 373 ($380$) GPa by LDA (GGA). The \textit{P6$_3$/mmc} phase, another candidate for a post-PPV phase, was not found in Al$_2$O$_3$ up to 700 GPa. No phonon softening was observed both in the CaIrO$_3$-type and in the U$_2$S$_3$-type phases at least up to 700 GPa. These phases should be dynamically stable within this pressure range. Figure 9 shows the calculated phase boundaries of Al$_2$O$_3$. The post-PPV transition between the CaIrO$_3$-type and the U$_2$S$_3$-type phases has a positive Clapeyron slope, as in NaMgF$_3$.

It is interesting to compare calculated compression curves and those obtained by dynamic compression experiments. In a shock experiment to ~340 GPa, no direct evidence of phase transitions was noticed, although there was an atypical relationship between shock velocity ($u_s$) and particle velocity ($u_p$): $u_s = C + Su_p$ in which $S$ (0.957) was unusually small, suggested a sluggish phase transformation (Erskine 1994). However, Figure 10 strongly suggests the presence of all three phases stable below 340 GPa in the raw data (Marsh 1980; Erskine 1994). With shock data alone, it is difficult to resolve phase transitions accompanied by small density changes comparable to the detectability limit.

The transition pressure to the U$_2$S$_3$-type polymorph of Al$_2$O$_3$ exceeds the pressure at the core-mantle boundary of the Earth (~135 GPa). Therefore the occurrence of U$_2$S$_3$-type Al$_2$O$_3$ should not alter the current views of the Earth’s lower mantle. However, it might affect our
Figure 8. Relative static LDA enthalpies of Al₂O₃ polymorphs with respect to the CaIrO₃-type phase (Umemoto and Wentzcovitch 2008). Dashed vertical lines denote corundum—Rh₂O₃(II)-type, Rh₂O₃(II)-type—CaIrO₃-type, and CaIrO₃-type—U₂S₃-type transition pressures, respectively.

Figure 9. Calculated phase boundary in Al₂O₃ (Umemoto and Wentzcovitch 2008). Solid and dashed lines denote LDA and GGA phase boundaries.
understanding of the rocky mantles of terrestrial exoplanets since it could change the solubility of Al₂O₃ in CaIrO₃-type MgSiO₃. The change in coordination number from 6 to 7 in the B site suggests that Al₂O₃ might ex-solve from MgSiO₃, in particular, from the smaller 6-fold B site occupied by silicon in the CaIrO₃-type structure. The ex-solution of Al₂O₃ could change electrical and thermal conductivities of the CaIrO₃-type solid solution, as the incorporation of Al₂O₃ changes the electrical conductivity of MgSiO₃ perovskite in presence of iron (Xu et al. 1998).

RARE-EARTH SESQUISULFIDES

Figure 11 shows crystal structures of rare-earth sesquisulfides, RR’S₃ (R, R’ = lanthanoid or actinoid), at 1000 °C and ambient pressure. In the RR’S₃ family of compounds, we find several structures (corundum, GdFeO₃-type (i.e., perovskite), CaIrO₃-type, and U₂S₃-type structures) adopted by MgSiO₃ and Al₂O₃. In addition, LaYbS₃ adopts the GdFeO₃-type structure at high temperatures and the CaIrO₃-type structure at low temperatures (Rodier et al. 1983; Mitchell et al. 2004), being reminiscent of the post-perovskite transition in MgSiO₃ with a positive Clapeyron slope (Murakami et al. 2004; Oganov and Ono 2004; Tsuchiya et al. 2004). Therefore a close relationship is suggested between the multi-Mbar crystal chemistry of planet-forming minerals and that of rare-earth sesquisulfides. The relationship between RR’S₃ crystal structures and cation radii can be summarized as follows: the corundum structure occurs with small R and R’ radii (e.g., Yb₂S₃), the U₂S₃-type structure with large radii, and the CaIrO₃-type and the GdFeO₃-type structures with large R in the A site and small R’ in the B site (e.g., NdYbS₃ and LaYbS₃). In R₂S₃, coordination numbers increase with increasing cation radii (Fig. 12). This chemical-pressure effect produces a sequence of structures consistent with that

**Figure 10.** LDA static pressure vs compression (solid lines) (Umemoto and Wentzcovitch 2008). ρ is the calculated density of each phase and ρ₀ is that of corundum at 0 GPa. Horizontal dashed lines represent calculated transition pressures: 82 GPa for corundum—Rh₂O₃(II)-type, 137 GPa for Rh₂O₃(II)-type—CaIrO₃-type, and 373 GPa for CaIrO₃-type—U₂S₃-type transitions. Experimental shock data are shown by black (Erskine 1994) and white circles (Marsh 1980). The black dashed line is a compression curve obtained by applying the Rankine-Hugoniot equations to a single linear fit to the Hugoniot data (Erskine 1994).
induced by pressure in Al$_2$O$_3$. Therefore, one might anticipate that rare-earth sesquisulfides are a set of low-pressure analogs of planet-forming silicates and oxides. RR’S$_3$ structures that do not occur in MgSiO$_3$ and Al$_2$O$_3$, may also play an important role. In fact, Al$_2$O$_3$ in the Gd$_2$S$_3$-type structure, one of the major structures of RR’S$_3$ compounds, and the U$_2$S$_3$-type polymorph have very similar enthalpies (see Fig. 8). The Ho$_2$S$_3$-type structure is also one of the likely forms between corundum and the Gd$_2$S$_3$-type/U$_2$S$_3$-type phases, but for Al$_2$O$_3$, this phase has very high enthalpy and is energetically unstable.

**M$_2$O$_3$ SESQUIOXIDES**

In addition to rare-earth sesquisulfides, M$_2$O$_3$ sesquioxides (M = trivalent cations: group IIIB metals, gallium and indium, and d-transition metals) could be another set of low-pressure analogs of planet-forming sesquioxides, especially of Al$_2$O$_3$. Ga$_2$O$_3$ and Fe$_2$O$_3$ exhibit sequences of phase transitions similar to Al$_2$O$_3$: corundum—Rh$_2$O$_3$(II)-type—CaIrO$_3$-type phases (in Ga$_2$O$_3$, corundum phase is preceded by the $\beta$ phase) (Shim and Duffy 2002; Ono et al. 2005; Ono and Ohishi 2005; Shim et al. 2009, Tsuchiya et al. 2007; Yusa et al. 2008a). The CaIrO$_3$-type structure occurs also in Mn$_2$O$_3$ (Santillán et al. 2006).

Caracas and Cohen (2007) performed a series of first-principles calculations of phase transitions in Al$_2$O$_3$, Ga$_2$O$_3$, Rh$_2$O$_3$, and In$_2$O$_3$. They showed that In$_2$O$_3$ should undergo the
Rh$_2$O$_3$(II)-type—CaIrO$_3$-type phase transition at a pressure as low as 47 GPa. In fact, In$_2$O$_3$ does not adopt quite exactly the CaIrO$_3$-type structure experimentally (Yusa et al. 2008b). The PPV transition pressures of Ga$_2$O$_3$ and Rh$_2$O$_3$ were estimated to be 136 GPa and ~360 GPa, respectively. This is contrary to expectations; The PPV transition pressure in Rh$_2$O$_3$ is expected to be lower than in Al$_2$O$_3$ and Ga$_2$O$_3$, since the ionic radius of rhodium (0.805 Å) is larger than those of aluminum (0.675 Å) and gallium (0.760 Å) (Shannon 1976). Results by Caracas and Cohen (2007) suggest the relationship between crystal structures in M$_2$O$_3$ and cation radii is not simple. Indeed, the sequence of phase transitions in M$_2$O$_3$ is very complex, as shown in Figure 13. This complexity indicates that $d$ electrons may be an important factor for determination of crystal structures. Partially-filled $d$ states have preferential orientations in space and should affect the positions of surrounding oxygens. Different $d$ states filling factors in different cations may give rise to a variety of crystal structures. For Fe$_2$O$_3$, magnetization is also an important factor (Shim et al. 2009). In the case of rare-earth sesquisulfides, the occupation of $f$ states varies, depending on the lanthanoid and actinoid atomic species. But the $f$ states are quite localized and should hardly affect chemical bonding. Therefore the relationship between crystal structure and ionic radii in rare-earth sesquisulfides is rather simple.

The Gd$_2$S$_3$-type structure is a high-pressure form of In$_2$O$_3$ and Sc$_2$O$_3$ with relatively large ionic radii (Yusa et al. 2008b, 2009). But, in In$_2$O$_3$ and Sc$_2$O$_3$, the enthalpy differences between the Gd$_2$S$_3$-type and the U$_2$S$_3$-type phases were found to be very small. In some experimental conditions, both phases might coexist, as in Dy$_2$S$_3$ (Meetsma et al. 1991). Ti$_2$O$_3$ is very
interesting. It transforms directly from corundum to the Th$_2$S$_3$-type phase (Nishio-Hamane et al. 2009). The Th$_2$S$_3$-type structure is isostructural with U$_2$S$_3$. The symmetry of both structures is $Pnma$. The difference between them is the type of orthorhombic distortion: lattice constant $a < c$ in the Th$_2$S$_3$-type and $a > c$ in the U$_2$S$_3$-type structure. So far, no transition from the CaIrO$_3$-type phase has been identified experimentally in M$_2$O$_3$. For Ga$_2$O$_3$, Mn$_2$O$_3$, and Fe$_2$O$_3$, whose highest-pressure forms identified experimentally appears to be the CaIrO$_3$-type structure (Shim et al. 2009), we anticipate post-PPV phases with U$_2$S$_3$-type or Th$_2$S$_3$-type structures.

**SUMMARY**

Two post-post-perovskite transitions have been identified by first principles calculations: the dissociation of MgSiO$_3$ into CsCl-type MgO and cotunnite-type SiO$_2$ at ~1 TPa and the transition of Al$_2$O$_3$ to the U$_2$S$_3$-type phase at ~370 GPa. These transition pressures are currently very challenging for static compression experiments. NaMgF$_3$ may be a good low-pressure analog of MgSiO$_3$. We found a close relationship between the multi-Mbar crystal chemistry of planet-forming minerals and that of rare-earth sesquisulfides. Some of them may be used as low-pressure analogs of planet-forming minerals in the multi-Mbar range. In transition-metal sesquioxides, the Th$_2$S$_3$-type phase (very similar to U$_2$S$_3$-type) was discovered in Ti$_2$O$_3$, suggesting that some transition-metal sesquioxides could serve as low-pressure analog(s) of Al$_2$O$_3$. Finally, a new type of dissociation, FeTiO$_3$ perovskite into (Fe$_{1-x}$,Ti$_x$)O and Fe$_{1+x}$Ti$_{2-x}$O$_3$, has recently been reported (Wu et al. 2009). This finding suggests that solid solutions (probably with transition-metals) among dissociation products (and in parent phases) could complicate the transition mechanisms we have discussed in this chapter.
ACKNOWLEDGMENTS

Many parts of this article are based on a set of first-principles studies which we performed using the Quantum-ESPRESSO distribution (Giannozzi et al. 2009). These works were supported by NSF grants No. EAR-0135533, EAR-0230319, ITR-0428774 (Vlab), EAR-0757903, EAR-0635990, and ATM-0428774 (Vlab).

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